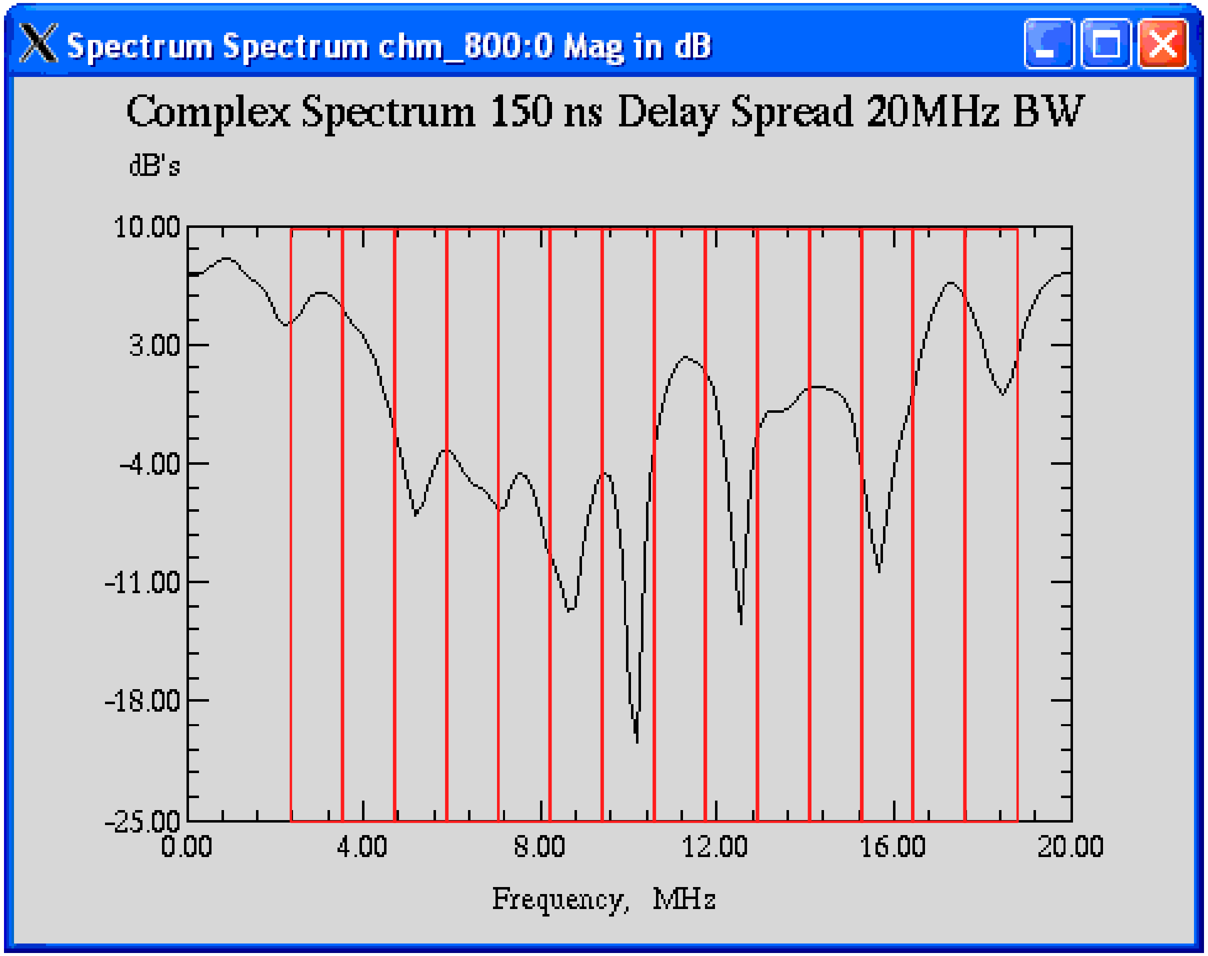
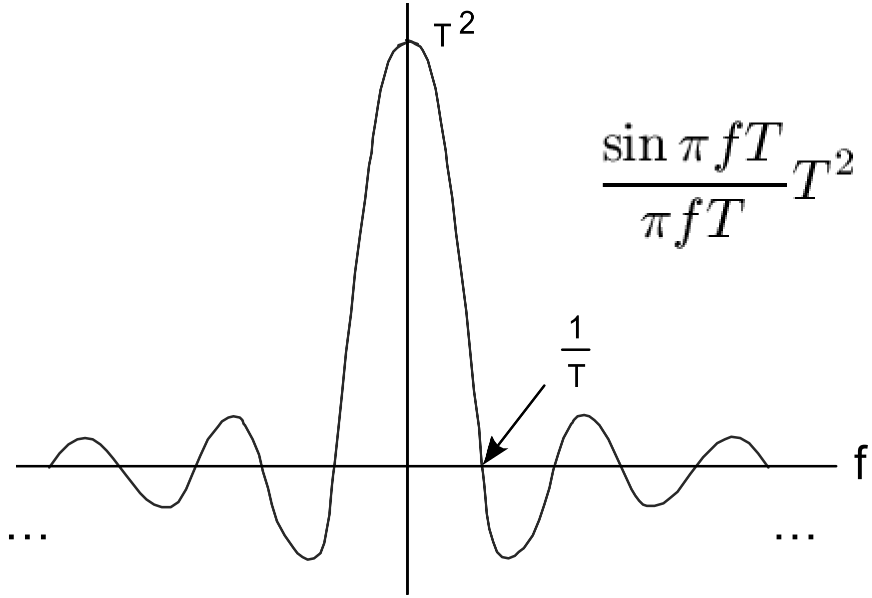
Modulation for Multipath

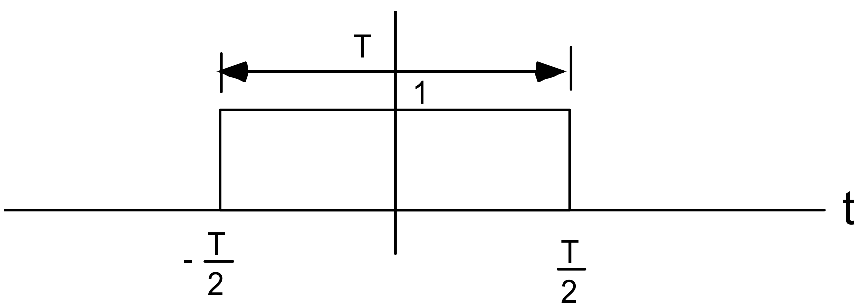
Orthogonal Frequency Division Multiplex (OFDM)

Silicon DSP Corporation

Multipath Channel: Multiple Sub Channels



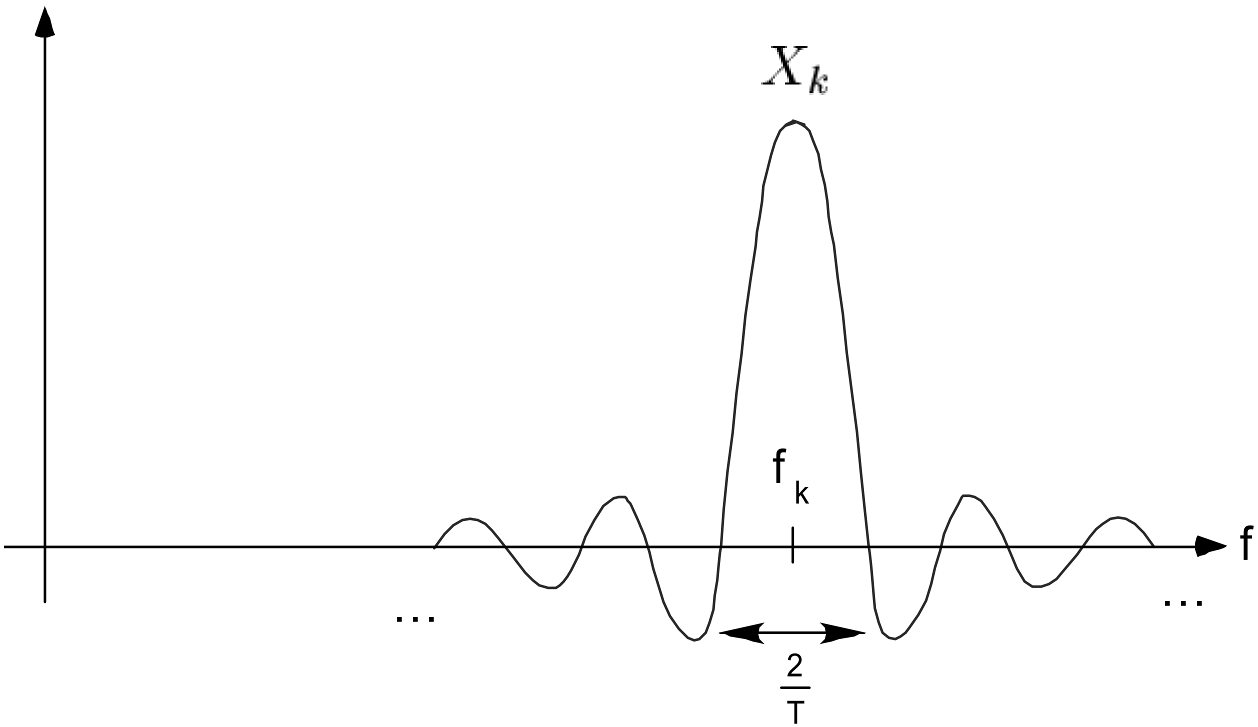


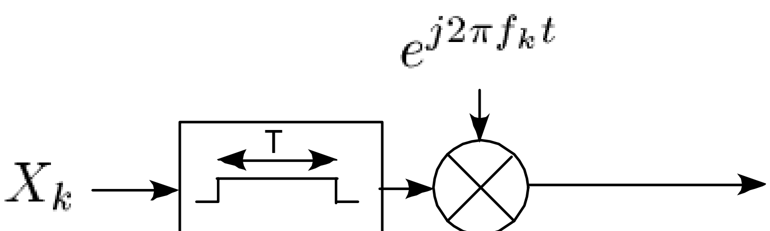


Fourier Transforms

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt$$

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi f t} df$$

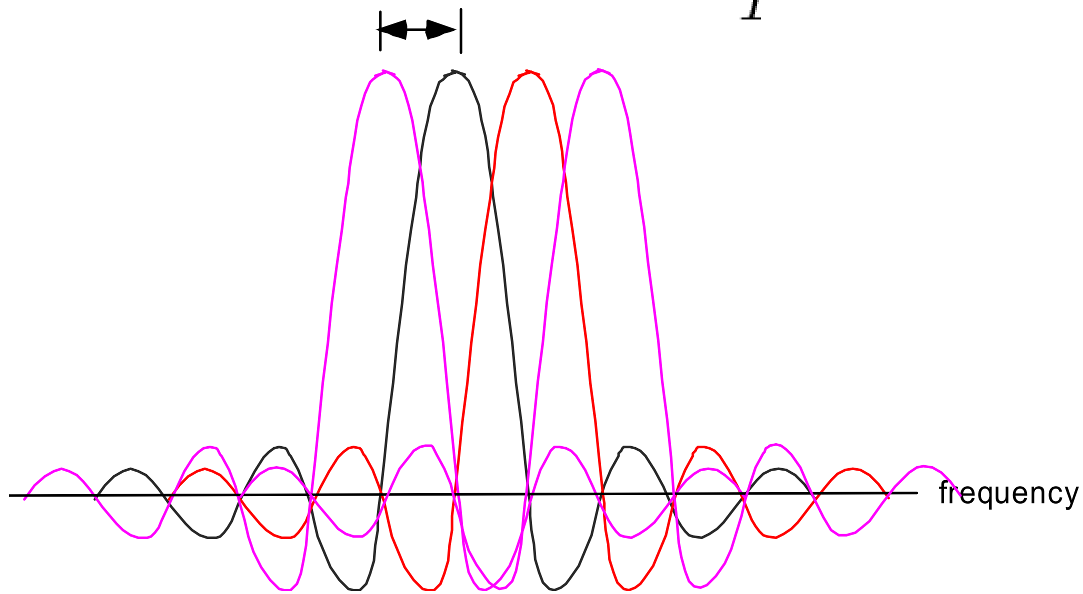


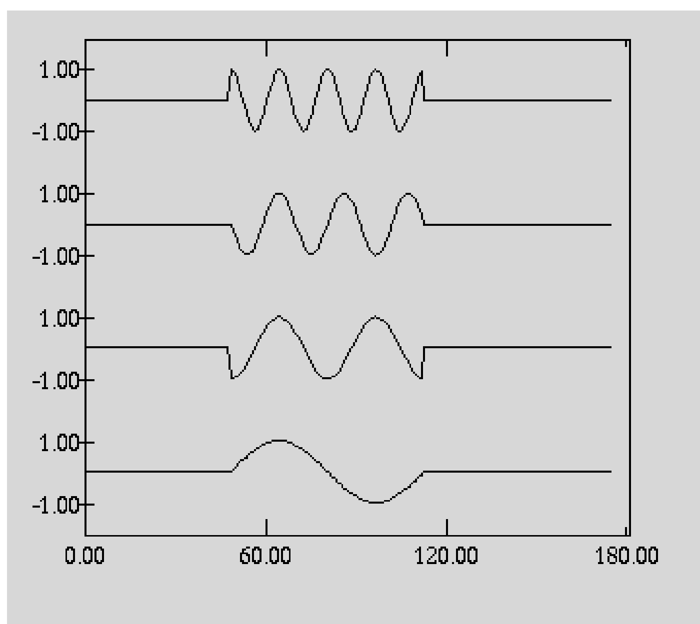


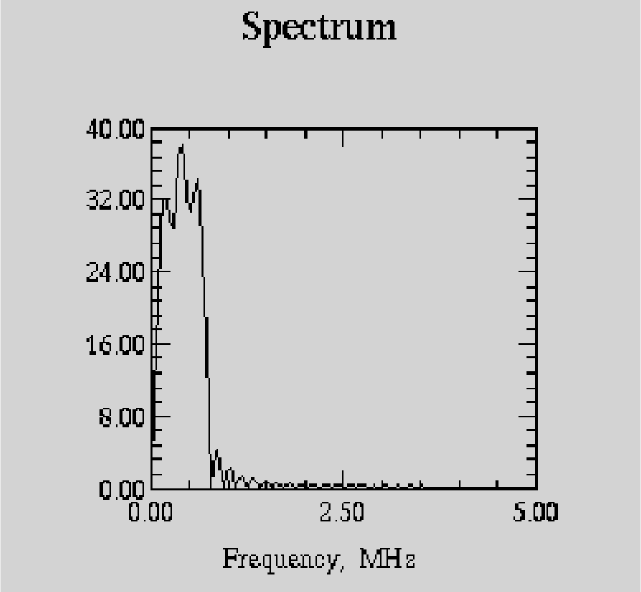
Modulation Property

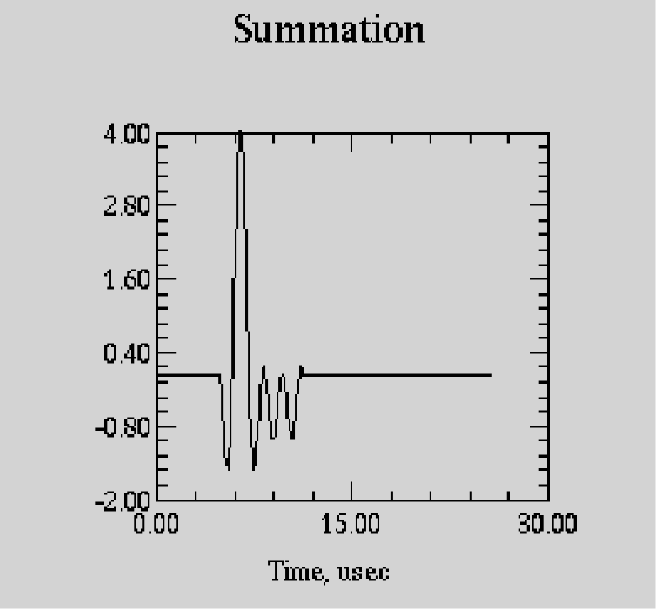
$$x(t)e^{j2\pi at} \leftrightarrow X(f-a)$$

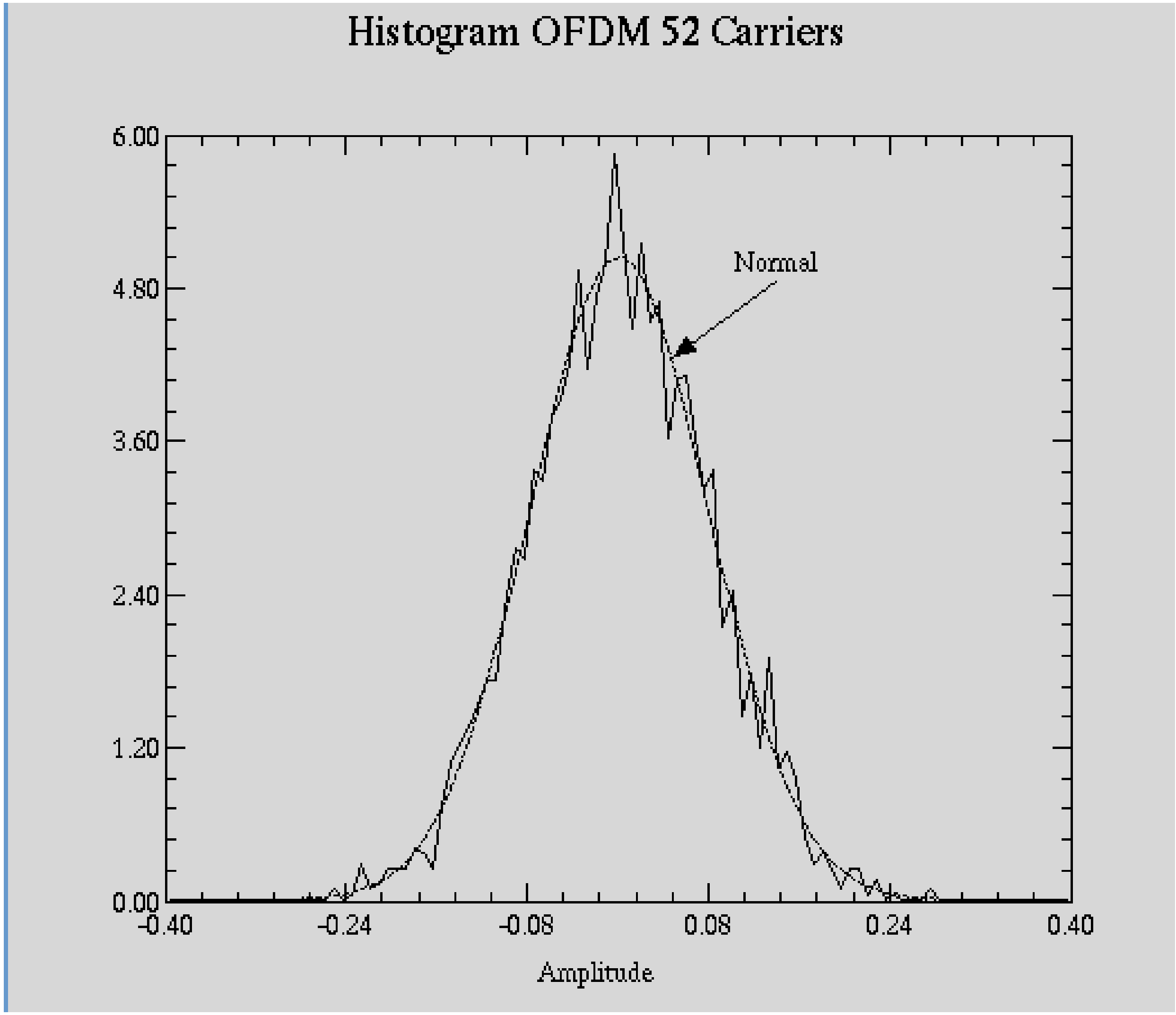
Carrier Spacing $\Delta f = \frac{1}{T}$



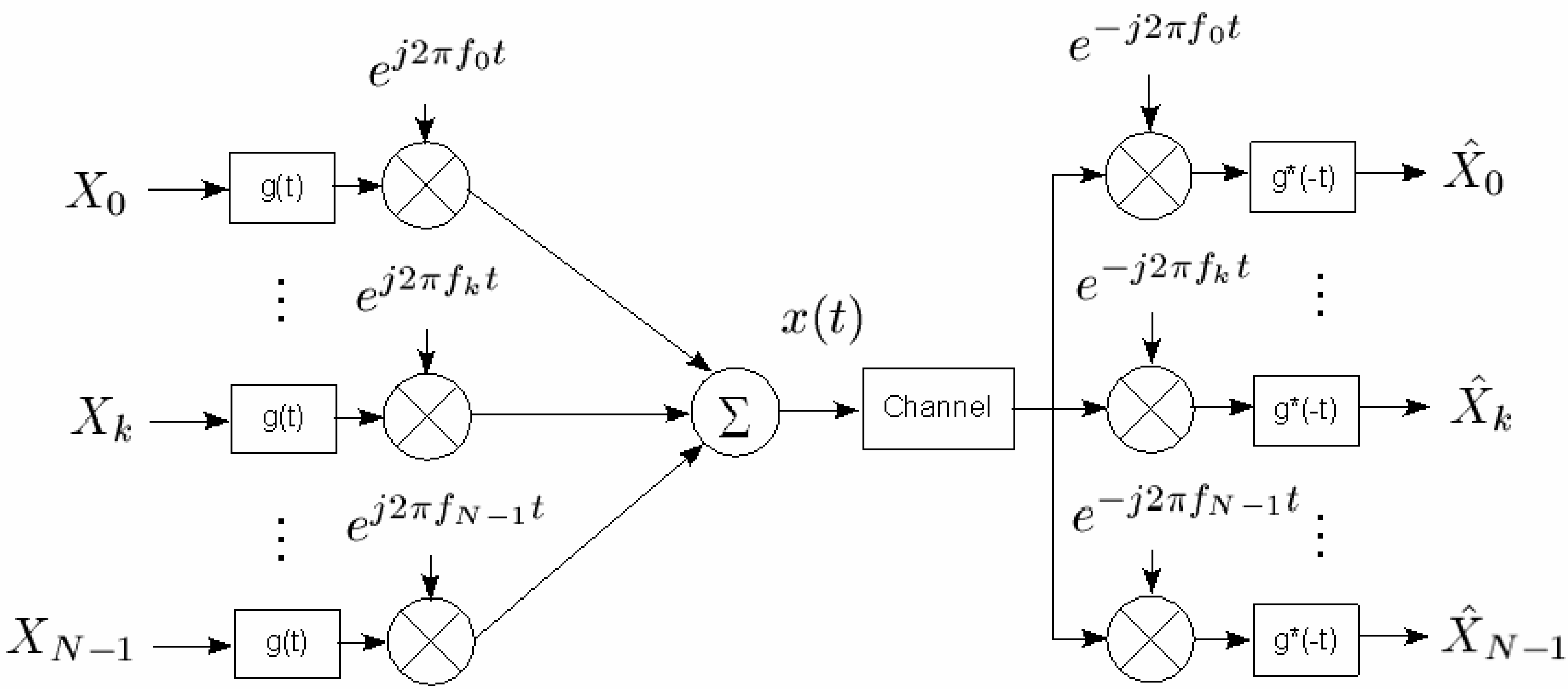



Spectrum
40.00
32.00
24.00
16.00
8.00
0.00
0.00
2.50
5.00
Frequency, MHz


Summation
4.00
2.80
1.60
0.40
-0.80
-2.00
0.00
15.00
30.00
Time, usec

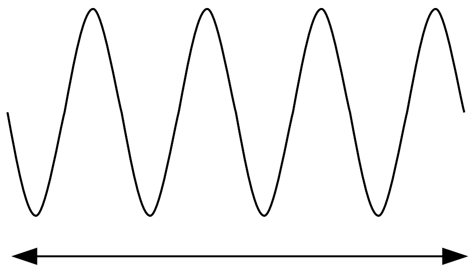


Histogram OFDM 52 Carriers
6.00
4.80
3.60
2.40
1.20
0.00
-0.40
-0.24
-0.08
0.08
0.24
0.40
Amplitude
Normal


X_0
X_k
X_{N-1}
g(t)
g(t)
g(t)
$e^{j2\pi f_0 t}$
$e^{j2\pi f_k t}$
$e^{j2\pi f_{N-1} t}$
Σ
$x(t)$
Channel
$e^{-j2\pi f_0 t}$
$e^{-j2\pi f_k t}$
$e^{-j2\pi f_{N-1} t}$
g*(-t)
g*(-t)
g*(-t)
$\hat{X}_0$
$\hat{X}_k$
$\hat{X}_{N-1}$

Derivation of DFT Formulation

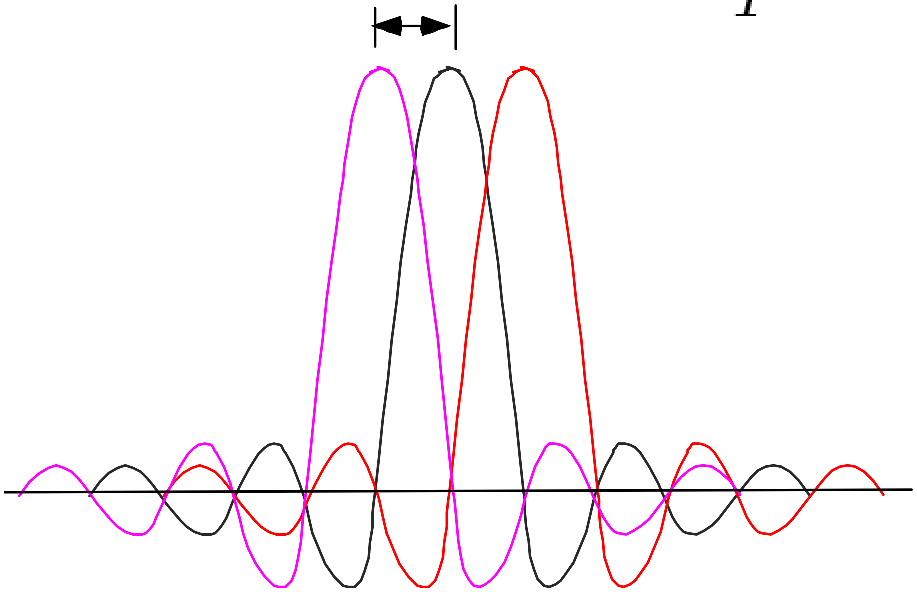
$$x(t) = \sum_{k=0}^{N-1} X_k e^{j2\pi f_k t}$$

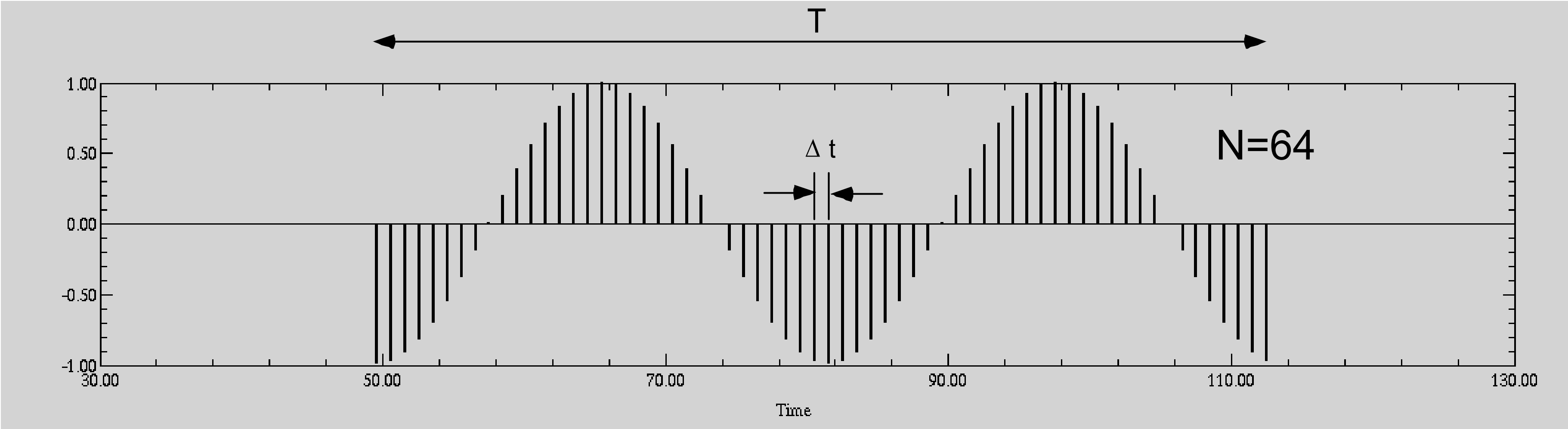


$$f_k = k\Delta f = \frac{k}{T}$$

Carrier Spacing $\Delta f = \frac{1}{T}$

$$x(t) = \sum_{k=0}^{N-1} X_k e^{j2\pi k \frac{t}{T}}$$





$$x_n = \sum_{k=0}^{N-1} X_k e^{j2\pi \frac{k}{T} n\Delta t}$$

$$t_n = n\Delta t$$

$$\Delta t = \frac{T}{N}$$

$$x_n = \sum_{k=0}^{N-1} X_k e^{j2\pi \frac{k}{T} \frac{T}{N} n}$$

$$x_n = \sum_{k=0}^{N-1} X_k e^{j2\pi \frac{k}{N} n}$$

Discrete Fourier Transform (DFT)

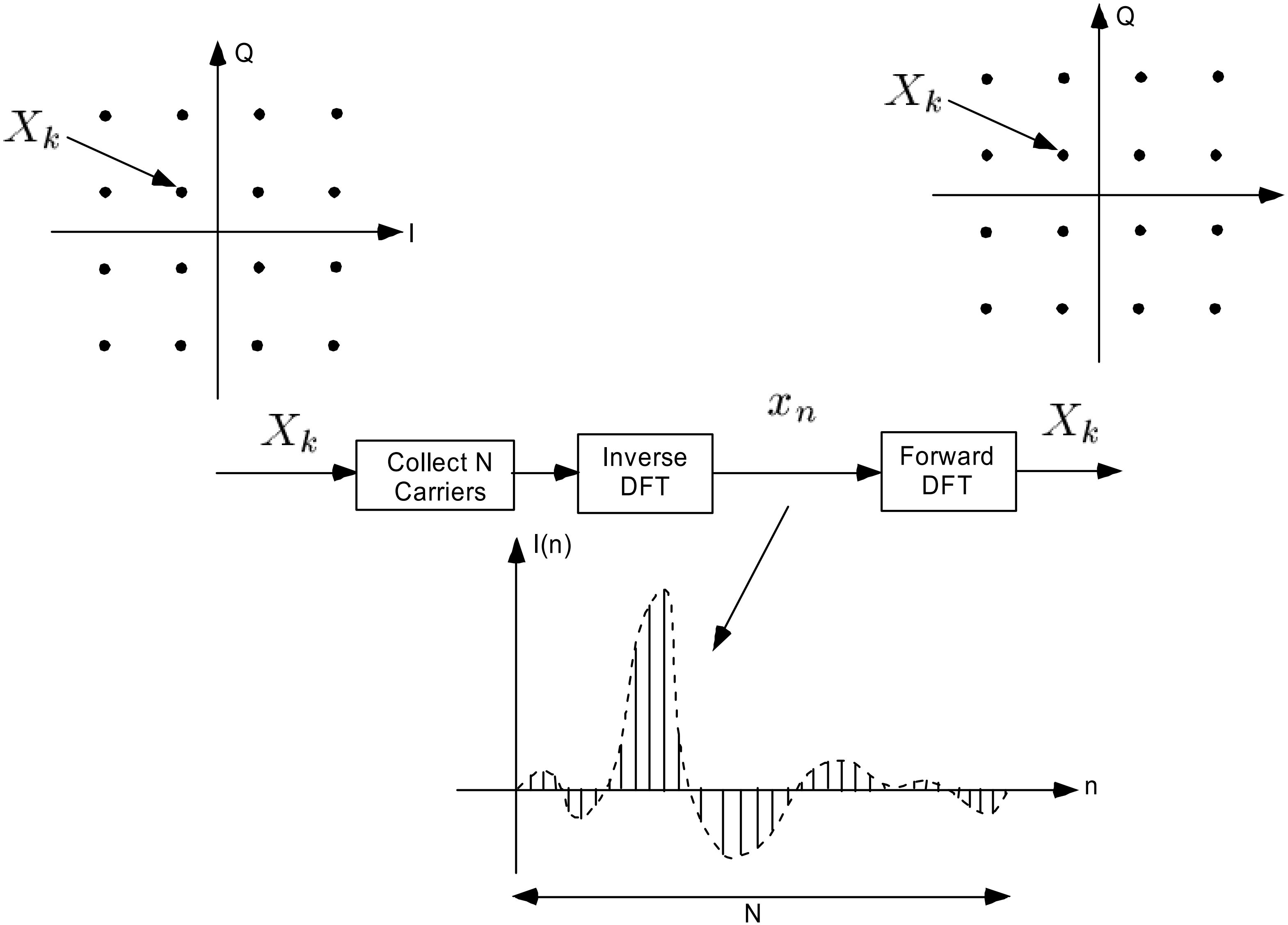
$$x_n = \sum_{k=0}^{N-1} X_k e^{j2\pi \frac{k}{N} n}$$

Definition of DFT

$$X_k = \sum_{n=0}^{N-1} x_n e^{-j2\pi \frac{k}{N} n}$$

Inverse of DFT

$$x_n = \frac{1}{N} \sum_{k=0}^{N-1} X_k e^{j2\pi \frac{k}{N} n}$$


Q
I
X_k
Q
I
X_k
X_k
Collect N Carriers
Inverse DFT
x_n
Forward DFT
X_k
I(n)
n
N

Matrix Formulation DFT

DFT $$X_k = \sum_{n=0}^{N-1} x_n e^{-j2\pi \frac{k}{N} n}$$

$$\underline{x} = \begin{bmatrix} x_0 \\ x_1 \\ \dots \\ x_{N-1} \end{bmatrix} \qquad \underline{X} = \begin{bmatrix} X_0 \\ X_1 \\ \dots \\ X_{N-1} \end{bmatrix}$$

$$\begin{bmatrix} X_0 \\ X_1 \\ \dots \\ X_{N-1} \end{bmatrix} = \begin{bmatrix} w^{00} & w^{01} & \dots & w^{0(N-1)} \\ w^{10} & w^{11} & \dots & w^{1(N-1)} \\ \dots & \dots & \dots & \dots \\ w^{(N-1)0} & w^{(N-1)1} & \dots & w^{(N-1)(N-1)} \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ \dots \\ x_{N-1} \end{bmatrix}$$

where w^{kn} are selected from the N roots of the unit circle $e^{-j\frac{2\pi}{N}}$,

$$w^{kn} = e^{-j\frac{2\pi}{N}kn}$$

$$\underline{X} = \mathbf{W}\underline{x}$$

Matrix Formulation Inverse DFT

DFT

$$\underline{X} = \mathbf{W}\underline{x}$$

$$X_k = \sum_{n=0}^{N-1} x_n e^{-j2\pi \frac{k}{N} n}$$

Inverse DFT

$$\underline{x} = \frac{1}{N}\mathbf{W}^{\mathbf{H}}\underline{X}$$

$$x_n = \frac{1}{N}\sum_{k=0}^{N-1} X_k e^{j2\pi \frac{k}{N} n}$$

Hermitian Transpose

$$\mathbf{W}^{\mathbf{H}}$$

Fast Fourier Transform (FFT)

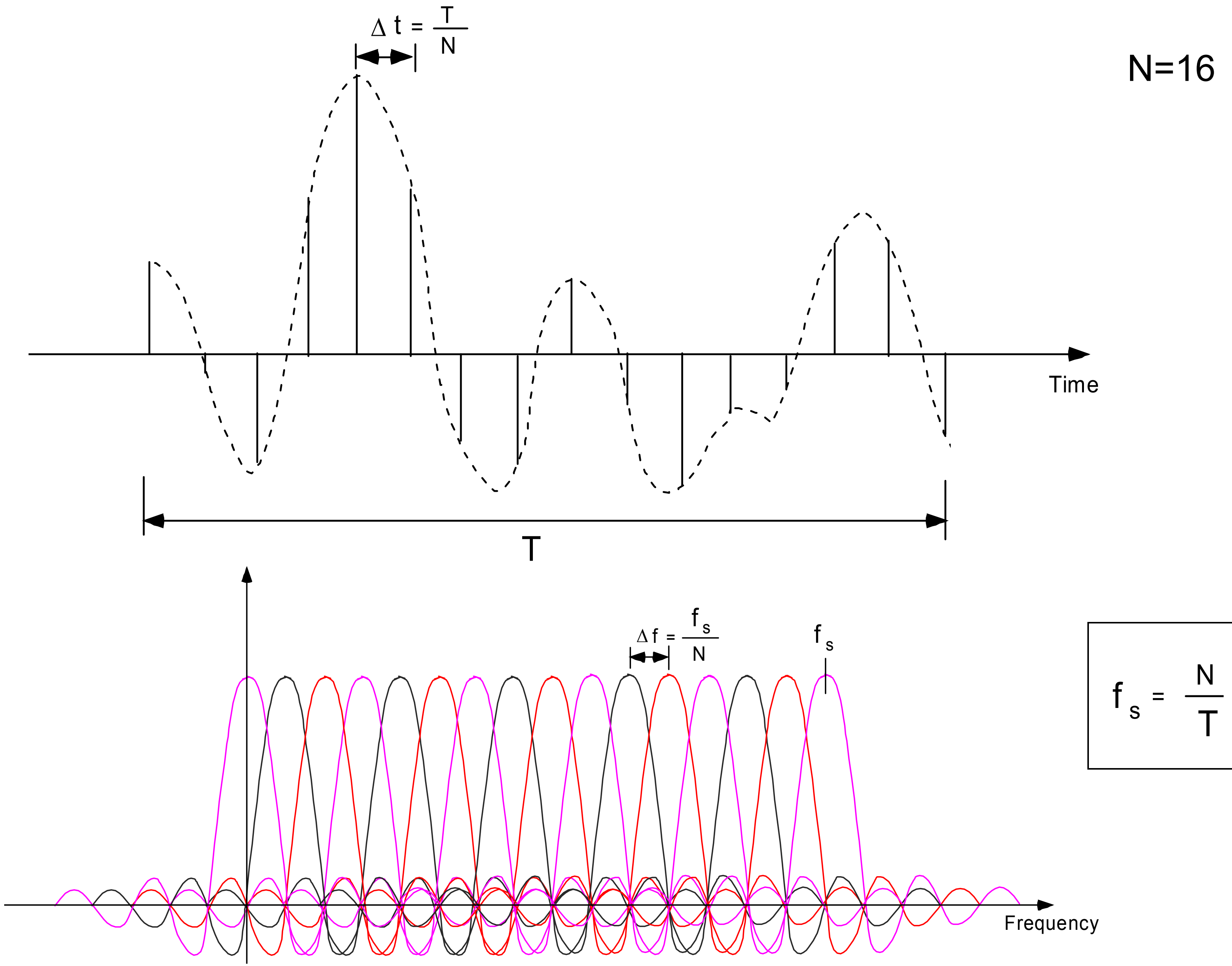
DFT and FFT Equivalent Mathematically (infinite precision)

DFT Requires Order N^2 Complex Multiplications

FFT Requires Order ($N \log_2 N$) Complex Multiplications

N	DFT	FFT	Reduction Factor
64	4096	384	10.7
256	65536	2048	32
1024	1048576	10240	102

OFDM and Sampling Rate



$$f_s = \frac{N}{T}$$

OFDM Example IEEE 802.11a

Bandwidth=20 MHz

From previous slide then,

f_s=20 MHz *that simple*.

Carrier Spacing determined by a number of factors.

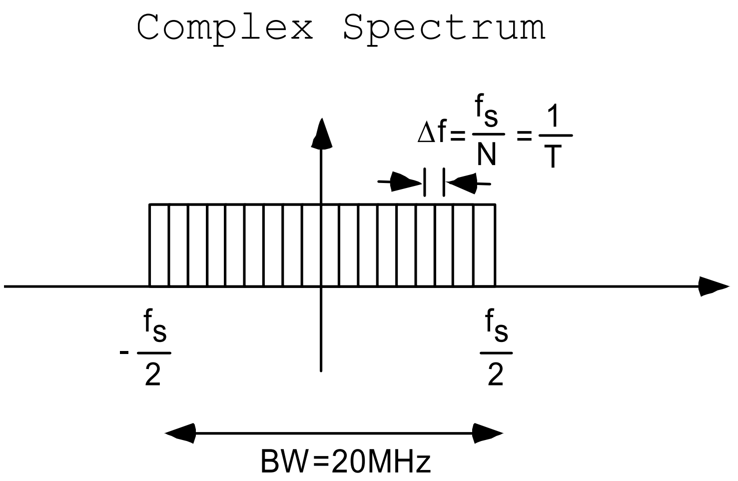
1. Avoid Frequency Selective Fading in each sub-channel
2. Keep FFT length reasonable
3. Satisfy overall throughput
4. Other factors (rates, robustness, carrier offset)

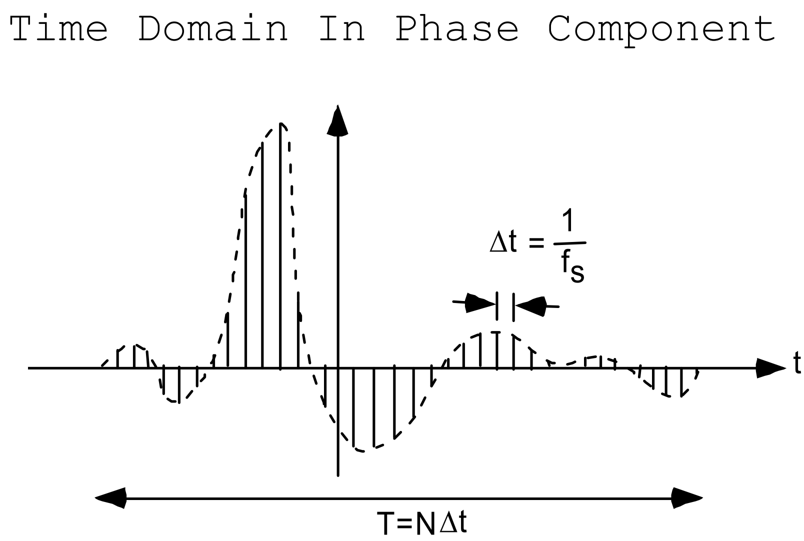
Let N=64. Then,

Carrier Spacing $\Delta f = f_s/N$= 20MHz/64=312.5 KHz

T= 1/Carrier Spacing =1/ Δf = 3.2 μsec

Tolerable Delay Spread roughly 1-2 μsec





OFDM Steady State Model

No Training / Acquisition/ Cyclic Prefix